Telecommunications Systems Exam

13 May 2024

Last name : SCIPER :

First name : —

Exam Rules

Please carefully read the following rules before starting the exam.

- This exam is a closed book, students may not use any paper or electronic resource;
- The only permitted resource is a 2-sided A4 page of personal handwritten notes;
- The only permitted materials are writing materials and a calculator;
- The exam sheets provide space to write the answers. If more paper is needed, blank pages will be distributed. Each additional page has to indicate the student's first and last name and SCIPER number;
- The exam duration is 2 hours, students shall immediately stop writing and close the exam sheet when the supervisor announces the end of the exam;
- When handing in the exam sheets until 15' before the end, students may silently leave the room;
- From 15' before the end of the exam, nobody is allowed to leave the room until the exam supervisor authorizes it;
- Any attempt of cheating or failure to comply to these rules will lead to a grade of 1.

Exam Tips

- Before starting the exam, read every question;
- If you are stuck on a question, go to another one and come back later;
- Keep an eye on the time;

Definitions

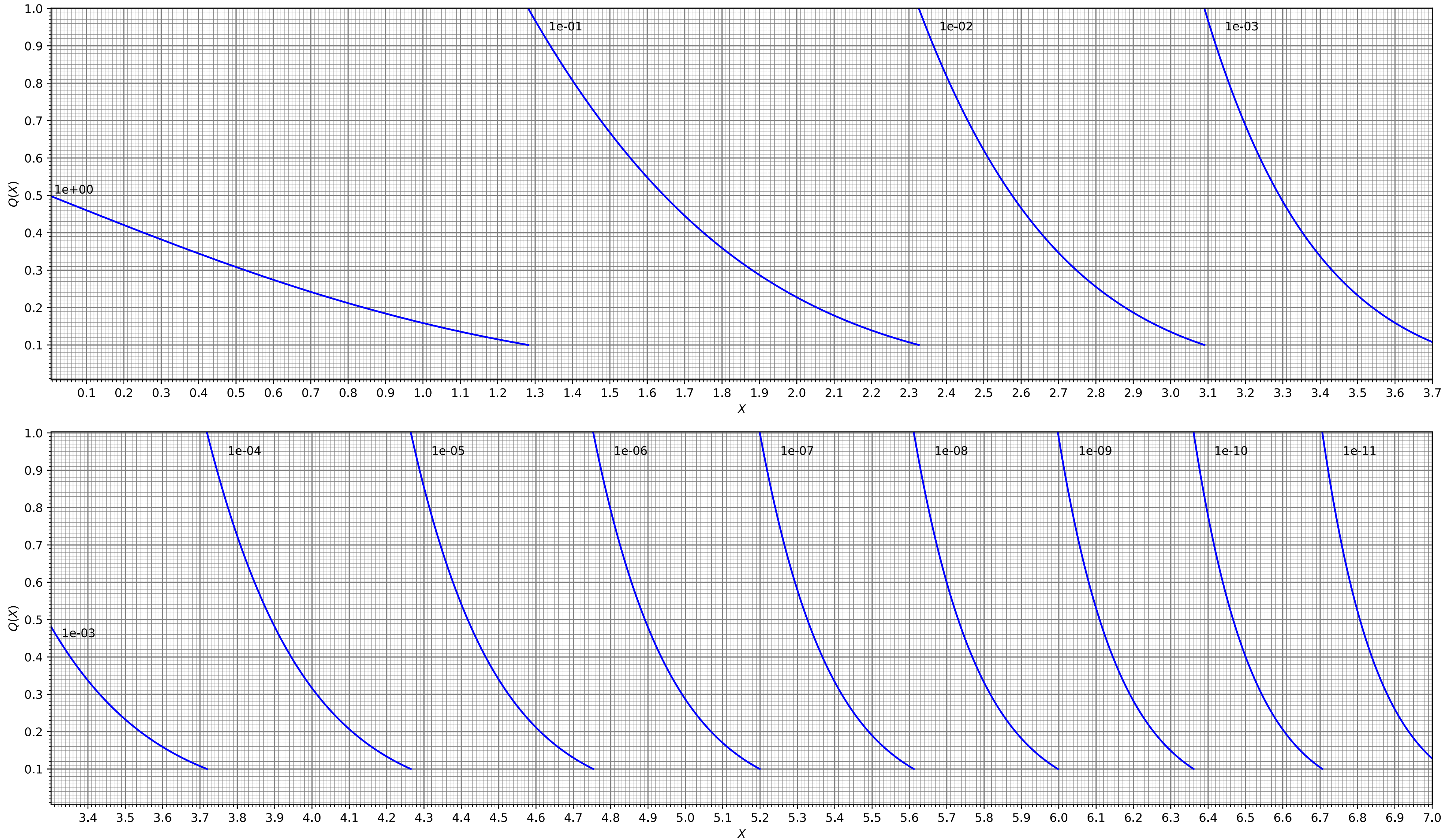
Throughout the exam, these are the definition of standard mathematical functions.

$$\Pi(t) = \begin{cases} 1, & \text{if } |t| \leq \frac{1}{2}, \\ 0, & \text{otherwise.} \end{cases}$$ rectangular function

$$\Lambda(t) = \begin{cases} 1 - 2|t|, & \text{if } |t| \leq \frac{1}{2}, \\ 0, & \text{otherwise.} \end{cases}$$ triangular function

$$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$ normalized sinc

Standard normal probability to be larger than X



1 Analog Modulation (5 pt.)

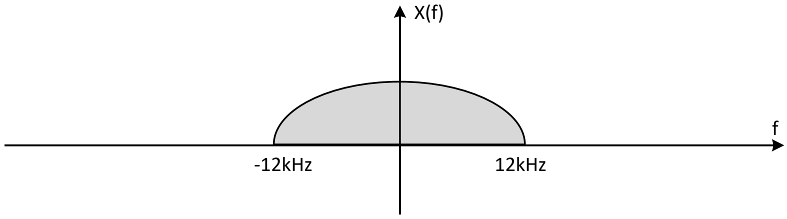


Figure 1: Frequency domain of $m(t)$.

We consider a baseband signal $m(t)$ shown in Figure 1, where $|m(t)|_{\max} = A$.

1. Given that the transmitted signal $m(t)$ has a one-sided baseband bandwidth of 12 kHz. Using coherent modulation, what is the minimum baseband bandwidth of the baseband filter to completely recover the signal if a transmitter and a receiver have a frequency offset $\Delta f = 2$ kHz?

2. Calculate the phase offset that causes a 3 dB attenuation. If there is more than one solution, you only need to specify one.

3. Let $s(t) = (A_c + \mu \cdot m(t)) \cos(2\pi f_0 t)$. If $\mu = 1$, what value should A_c take to enable non-coherent de-modulation at the receiver?

4. For $\mu = 0.7$, calculate the power efficiency.

5. Assume that $m(t) = \cos(30\pi \cdot 10^3 t)$ and a noise power spectrum density $N_0 = -105$ dBm/Hz. If $\text{SNR}_o = 50$ dB and the path attenuation is also 50 dB, what is the required transmitted power P_T in dBm?

2 Sampling, Quantization and Information Theory (10 pt.)

A digital PCM signal must be transmitted through a wire with analog amplifiers before being regenerated by a digital repeater. The wire is composed of two segments of attenuation $\alpha = 6\,$dB and maximum bandwidth of $B = 150$ kHz in the baseband. Both wire segments are terminated by analog amplifiers of gain $G = 3\,$dB. Each amplifier introduces some additional white noise of power $P_N = 5\ \mu$W. The input signal has a power of $P_{in} = 38\,$dBm and an SNR of SNR$_{in} = 63\,$dB

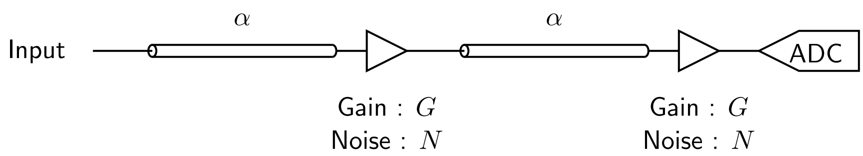


Figure 2: Transmission line.

1. Compute the signal power P_{s-ADC}, the noise power P_{n-ADC} and the SNR at the input of the ADC.

2. Compute the signal-to-quantization-noise ratio SQR for $Q = 4$ bit;

3. Compute the max Q if we want to keep $SQR \leq SNR$;

4. What is the channel capacity in [bits/s]?

5. Assume that the PCM signal at the input is transmitted at Baud rate $f_s = 250$ kBd and pulse-shaped by a raised cosine with rolloff factor $\beta = 0.12$. What is the minimum cut-off frequency of the ADC (assuming a perfect low-pass filter)?

3 Digital Modulation and Error Rate (5 pt.)

We are interested in an M-PAM constellation where the constellation points may have an offset $\alpha \geq 0$, i.e., $\mathcal{O}_M = \{\pm\frac{d}{2} + \alpha, \pm 3\frac{d}{2} + \alpha, \ldots, \pm(M-1)\frac{d}{2} + \alpha\}$

1. If $\alpha = 0$, what is the SNR penalty from 4-PAM to 64-PAM with the same symbol error rate?

2. For $\alpha = +\frac{3}{2}d$ without phase offset, derive the symbol error rate as a function of M and the SNR under an additive white Gaussian noise channel (AWGN).

3. Consider a system with 4-PAM ($\alpha = 0$) that has a noise power spectral density of $N_0 = -105$ dBm/Hz. We would like to get to an error rate of $\epsilon = 10^{-5}$, what is the required E_b/N_0? (Hint: $Q(\sqrt{18.96}) \approx 6.6 \times 10^{-6}$)

4. If the received power $P_R = -20$ dBm, what is the corresponding bit duration and required signal

baseband bandwidth with an ideal SINC filter?

5. What is the spectrum efficiency in subquestion (4)?

4 Design of a Telecommunication System (15 pt.)

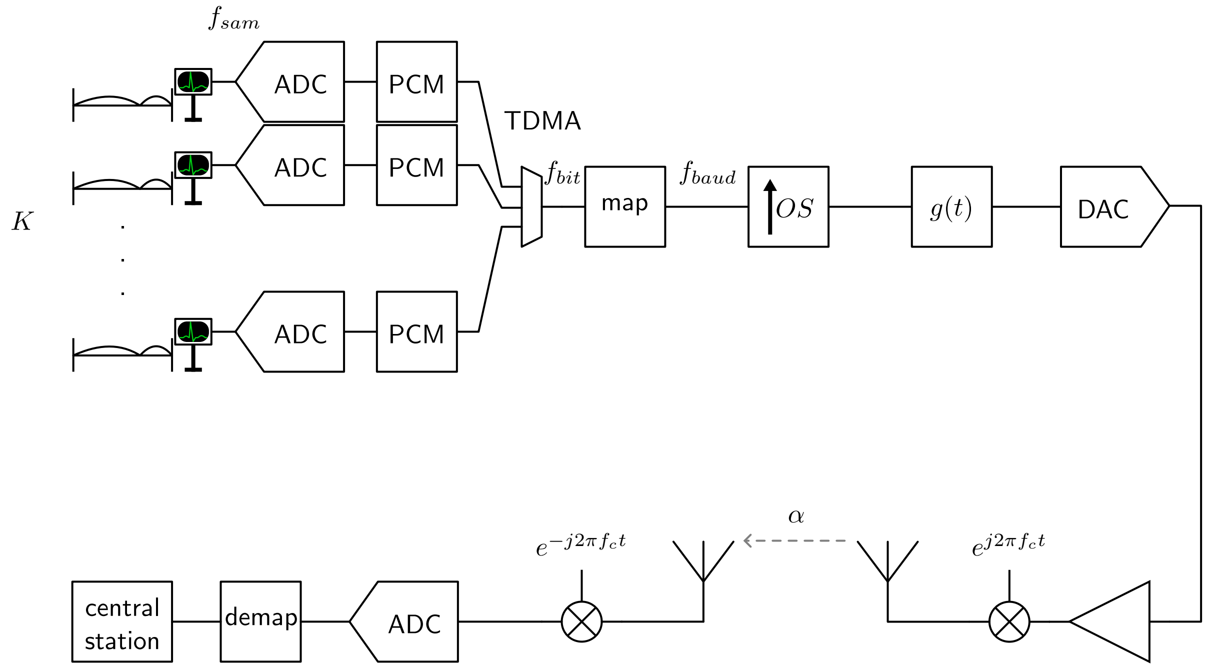
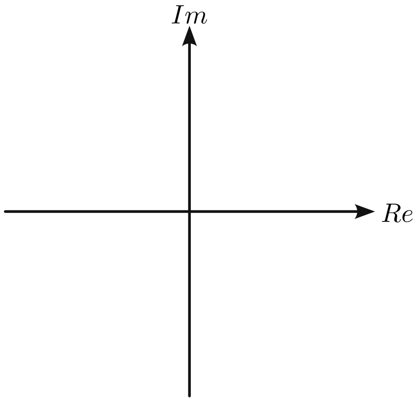


Figure 3: Block diagram of the transmission system.

All the $K = 35$ beds of the first floor of a hospital are equipped with an electrocardiogram monitor that measures the electrical signal from the patient's heart. This signal is sampled at frequency f_{sam}, quantized and transmitted by binary PCM to a multiplexer along with all the signals from the other beds. The K signals are multiplexed using TDMA and mapped on an 8-PSK constellation before being transmitted through a wireless channel to the central station of the hospital. Upon reception, the signal is demapped and fed to the central station.

1. Each bed is equipped with the same electrocardiogram device. The measured signal has a maximum amplitude of $A = 5\,$mV and a maximum frequency of $f_{ECG} = 100\,$Hz. Knowing that we want a maximum quantization error of $e = 1.25\,\mu$V and a sampling frequency that is 25% above the Nyquist rate, calculate the minimum number of quantization bits Q required and the sampling frequency f_{sam}.

2. Once multiplexed, the signal is mapped on an 8-PSK constellation. Sketch the unit-power constellation on the axes below and suggest an associated binary code that minimizes the bit errors should a demapping error occur. Then, calculate the bit rate f_{bit} at the output of the multiplexer and the Baud rate f_{baud} at the output of the mapper.



3. Before being transmitted, the mapped signal must be pulse-shaped to occupy a limited frequency band. This system uses a raised-cosine pulse-shaping filter $g(t)$ with roll-off factor β. We know that the wireless channel on which the signal is to be transmitted only has a bandwidth of $B_c = 50\,$kHz <u>in passband</u>. Calculate the roll-off factor to ensure that the bandwidth of the transmitted signal fits in the channel passband with a margin of 15%.

4. The pulse-shaping filter has 21 taps (i.e., the number of samples the FIR filter is made of). In order for the pulse to spread over the current symbol plus the two symbols after and the two symbols before, calculate the oversampling factor OS (<u>Hint</u>: OS must be an integer, visualize the spread of the pulse by sketching it over the oversampled symbols).

5. The pulse-shaped signal is then converted to the analog domain, up-converted to passband and amplified to be transmitted with a power of $P_{tx} = 20\,$dBm. The channel attenuates the signal by $\alpha = 2.8\,$dB and at the receiving side, a noise of power $P_N = -10\,$dBm is picked up over the whole channel band B_c. Assuming that the components of the receiver do not introduce any noise, what is the energy-per-bit to noise ratio E_b/N_0 at the receiver ?

6. For an M-PSK constellation, the probability of error can be approximated by :

$$P_e \approx 2Q\left(\sqrt{\frac{2E_b\log_2(M)}{M^2 N_0}}\right)$$

Where E_b is the energy per bit. Calculate the probability of error at the 8-PSK demapper. For a packet size of $L = 100'000$ bits, calculate the packet error rate (use the Gaussian abacus provided with the exam).

7. In order to cope with the error rate, we want to introduce an error detection mechanism to send an automatic repeat request (ARQ) when an error is detected. To make sure that the system can keep up with the data measured by the electrocardiograms, we must ensure that not too much redundancy is introduced. Fortunately, we have a margin of 15% on the available bandwidth, which means that, if we use the full bandwidth, we can tolerate a relative goodput of $\Theta = 85\%$. For a package length of $L = 100'000$ bits, calculate the number of bits O that the redundant part of each packet can contain.

Can we do the same for $L = 400'000$ bits? Explain why with your own words.